

System Modelling and Design

COMP2111



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+ tutors:

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Formal

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System Modelling and Design COMP2111

Credit for the material
also goes to:

Paul Hunter,
Christine Rizkallah,
Liam O'Connor,
and Carroll Morgan



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Zhuo (Zoey) Chen
Raphael Douglas Giles

We'll learn to

model systems in a way that's unambiguous and mathematically precise.

We'll be able to

say what it means for a system to satisfy its specification,
and prove that it does so.

We'll need

a substantial toolbox of discrete math and formal logic.

Don't worry; we'll teach it, not assume it.

Non-examples

September 1981

Transmission Control Protocol
Functional Specification

3.9. Event Processing

The processing depicted in this section is an example of one possible implementation. Other implementations may have slightly different processing sequences, but they should differ from those in this section only in detail, not in substance.

The activity of the TCP can be characterized as responding to events. The events that occur can be cast into three categories: user calls, arriving segments, and timeouts. This section describes the processing the TCP does in response to each of the events. In many cases the processing required depends on the state of the connection.

Events that occur:

User Calls

OPEN
SEND
RECEIVE
CLOSE
ABORT
STATUS

Arriving Segments

SEGMENT ARRIVES

Timeouts

USER TIMEOUT
RETRANSMISSION TIMEOUT
TIME-WAIT TIMEOUT

The model of the TCP/user interface is that user commands receive an immediate return and possibly a delayed response via an event or pseudo interrupt. In the following descriptions, the term "signal" means cause a delayed response.

Error responses are given as character strings. For example, user commands referencing connections that do not exist receive "error: connection not open".

Please note in the following that all arithmetic on sequence numbers, acknowledgment numbers, windows, et cetera, is modulo 2^{32} the size of the sequence number space. Also note that " $=<$ " means less than or equal to (modulo 2^{32}).

Non-examples

This RFC is a specification in English.

Natural language specs tend to have:

- Ambiguities
- Room for interpretation
- Important details in the writer's head absent from actual text.

September 1981

Transmission Control Protocol
Functional Specification

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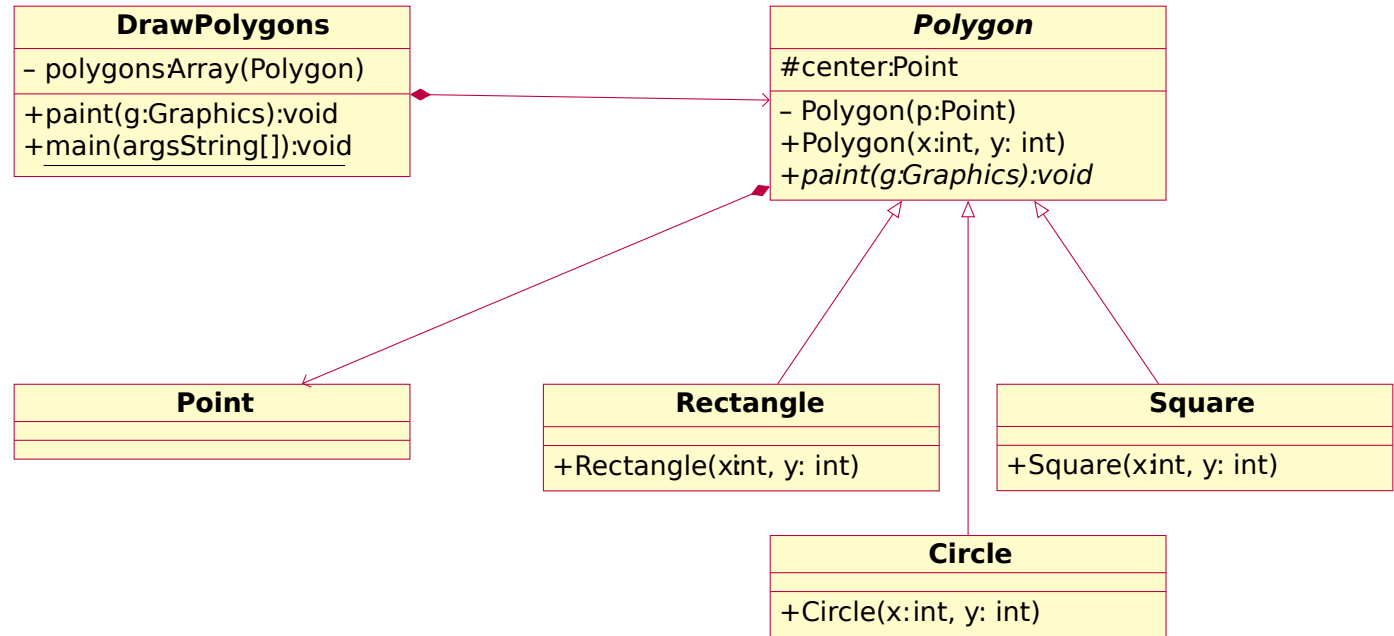
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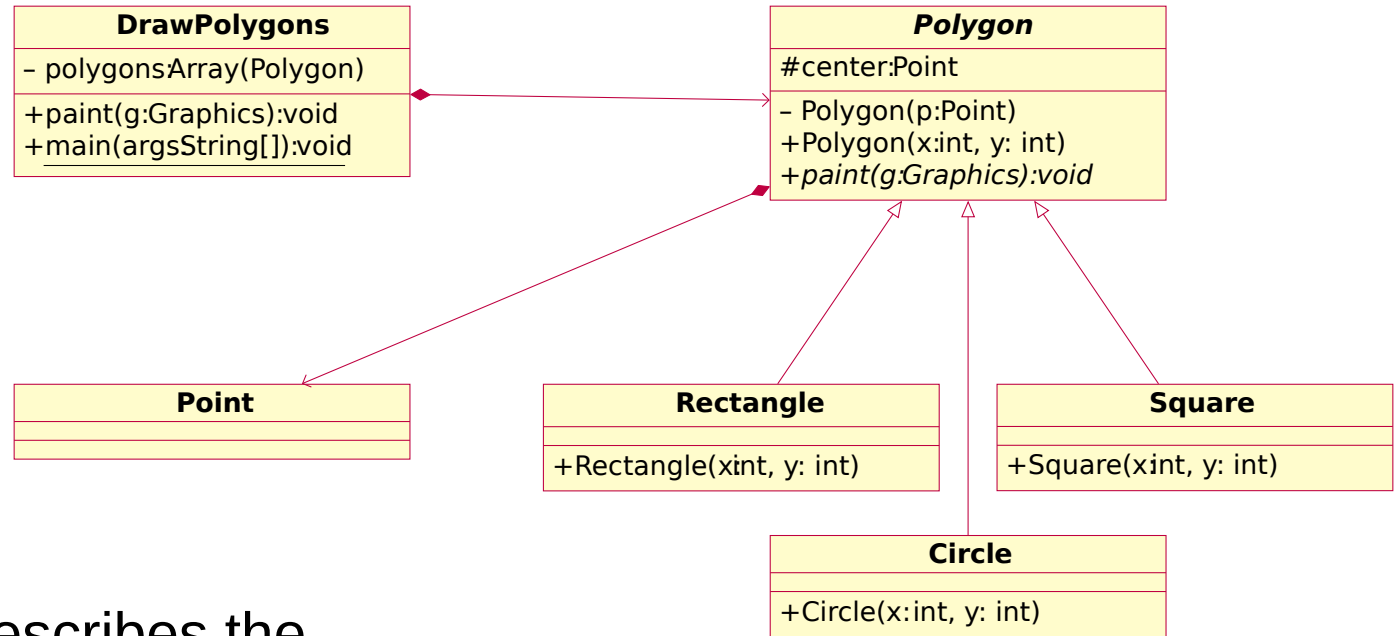
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Non-examples



Non-examples



This UML diagram describes the *structure* of the system, not its behaviour.

Resources

Course website: <http://www.cse.unsw.edu.au/~cs2111>

- Lecture slides, tutorials
- Assignment instructions
- ...

Ed forum: <https://edstem.org/au/courses/15105/>

- General announcements
- Class discussion, announcements
- E-mail cs2111@cse.unsw.edu.au if you haven't been invited!

Moodle: <https://moodle.telt.unsw.edu.au/>

- Lecture recordings
- Weekly quizzes

Examination

- Weekly quizzes: 15 credits total
 - After the lectures of every week (except W6 and W10).
 - Will appear on Moodle.
 - Deadline: Monday 4PM (before start of next week's lectures)
- Three assignments (individual or pair, written): $11+12+12=35$ credits
- Final exam (online, format TBA): 50 credits

Introduction to “Formal” Logic

Start here



This is a (draft) textbook for
COMP6721 (In-)Formal
Methods by Carroll Morgan

It's on the course website.

D	The arithmetic of conditions	207
D.1	Introduction and rationale	207
D.2	Why is my program correct?	207
D.3	How do I write my program in the first place?	209
D.4	Calculating with conditions	210
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D.14	(General) formulae	223
E	Some helpful logical identities	225
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Introduction to “Formal” Logic

element

set

intersection

membership

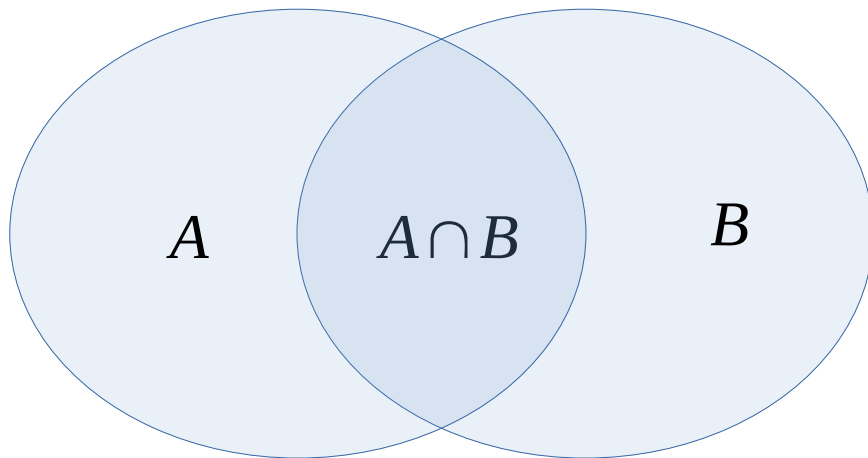
$$x \in S$$

$$x \notin S$$

$$x \in A \cap B$$

defined?

Introduction to “Formal” Logic



$x \in A \cap B$

if and only if

$x \in A$

 $x \in B$

Introduction to “Formal” Logic

intersection $A \cap B$

union $A \cup B$

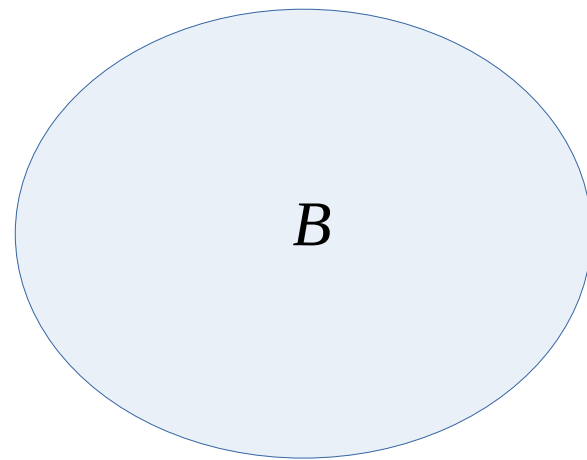
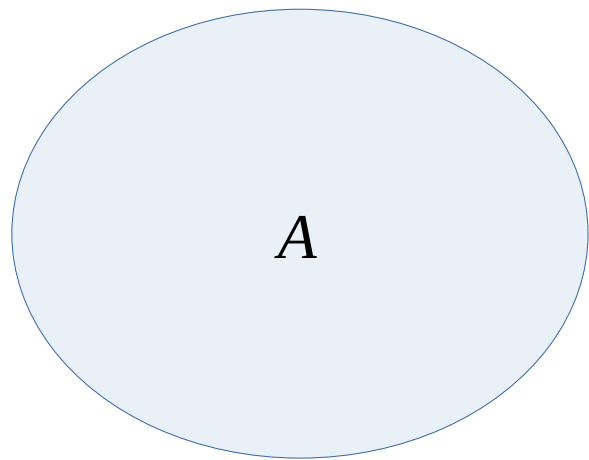
subset $A \subseteq B$

Q: Subset is not like others
in an important way. How?

$x \in A \cup B$ if and only if $x \in A$ $x \in B$

$A \subseteq B$ if and only if $x \in A$ $x \in B$

Introduction to “Formal” Logic



Where is $A \cap B$ now?

Introduction to “Formal” Logic

Let's prove $A \subseteq A \cup B$

Why so pedantic?

$$\{y \mid y \subseteq x\} \quad \underline{\hspace{1cm}}$$

$$\{y \mid y \in x\} \quad \underline{\hspace{1cm}}$$

Why so pedantic?

$x \in x$ _____

Does it make sense to write?

Is it ever true?

$\{ x \mid x \in x \}$ _____

Why so pedantic?

$$\{ x \mid x \notin x \}$$

$$y = \{ x \mid x \notin x \}$$

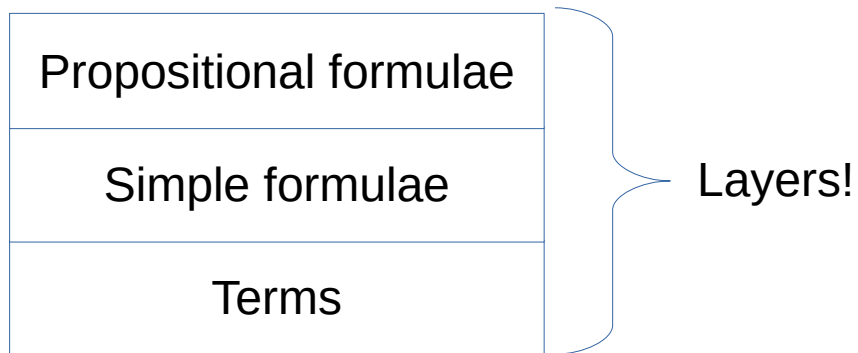
$$y \in y \Rightarrow y \notin y$$

$$y \notin y \Rightarrow y \in y$$

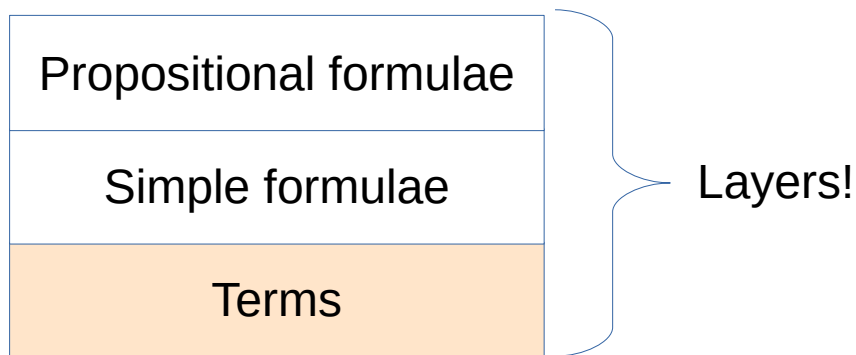
“There is just one point where I
have encountered a difficulty”
- Bertrand Russell

Q: Why does this matter?

The language of logic



The language of logic

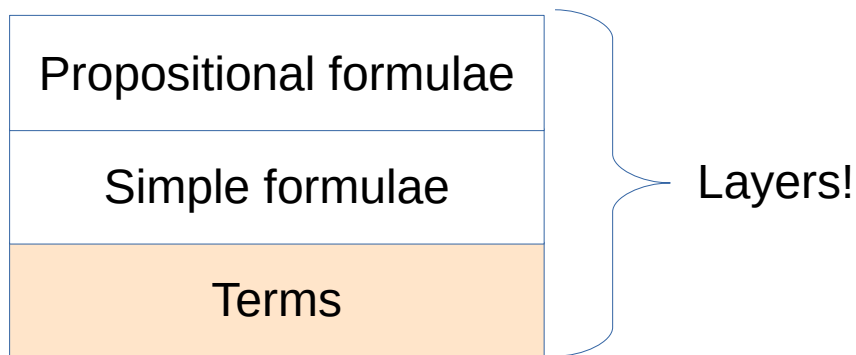


A *term* is either

- (a) a variable, or
- (b) a constant symbol, or
- (c) a function symbol applied to the correct number of other terms.

A function's number of arguments is its *arity*.

The language of logic



A *term* is either
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A function's number of arguments is its *arity*.

variables

x, y, \dots

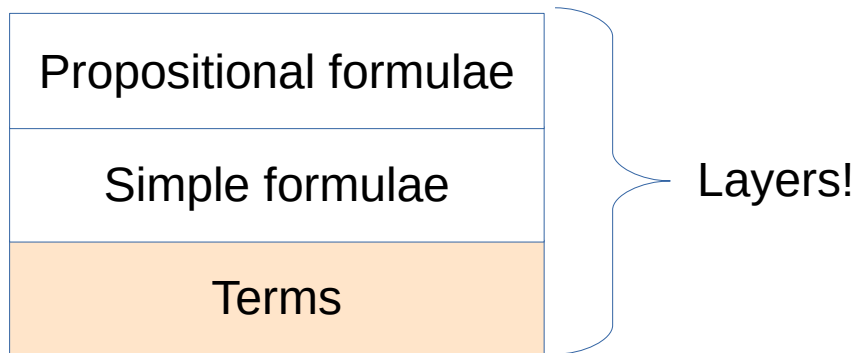
Constants

$1, \{\}, \pi$

functions

element, subset,
 $+, -, !$

The language of logic

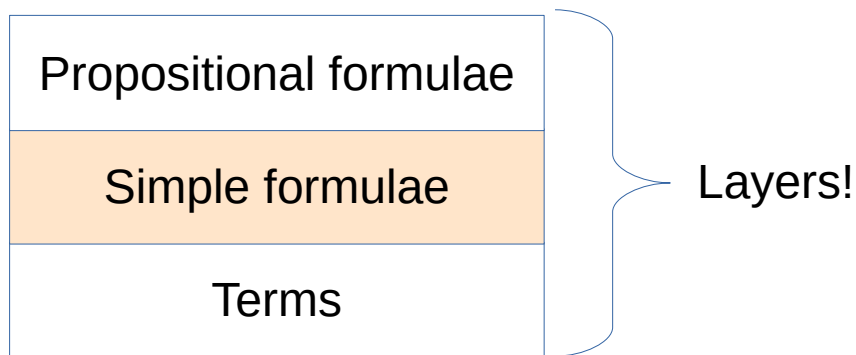


Terms have *values*

$x+1$	$\sin(x/2)$
xy	$x+$
0	$x++$
x	

Not terms

The language of logic



A *simple formula* is a predicate symbol applied to the correct number of (term) arguments.

$t < u$

$t = u$

$t \geq u$

$even(t)$

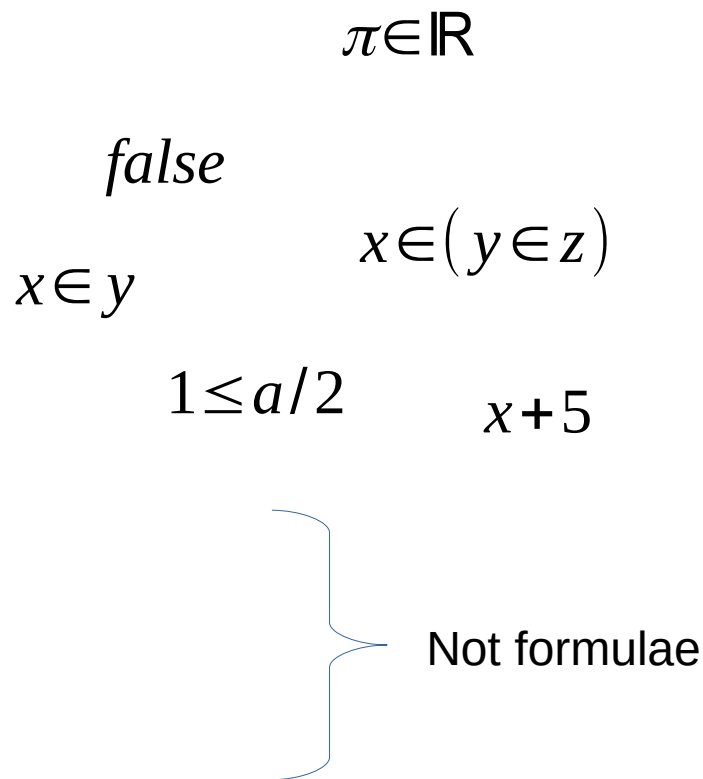
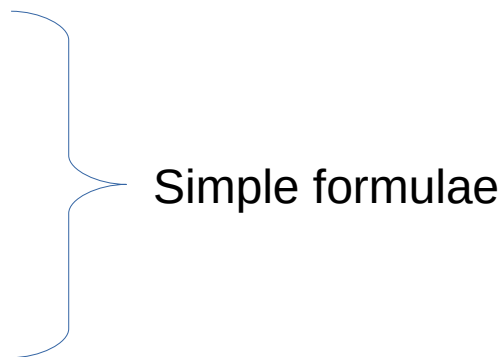
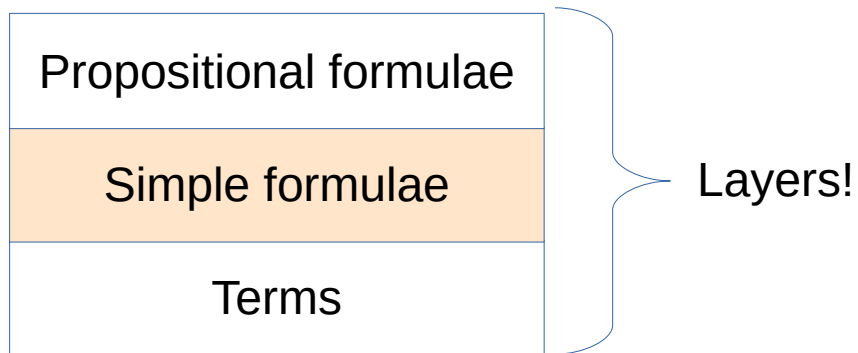
$t \in u$

$false$

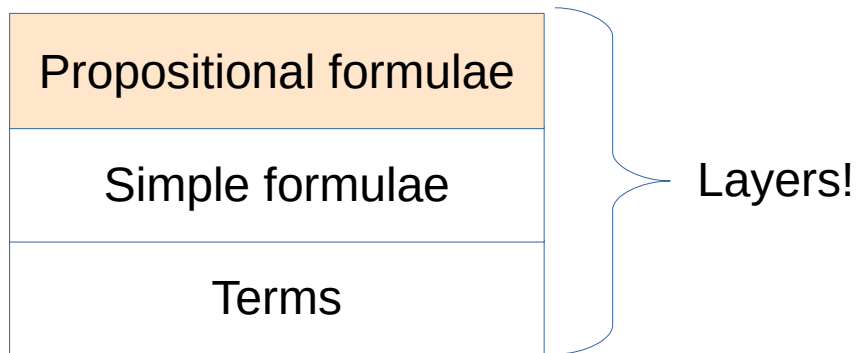
t, u are terms

These can be True or False

The language of logic



The language of logic

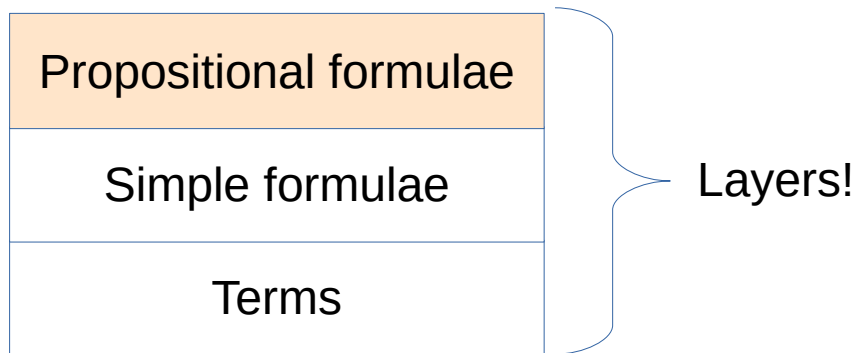


A *propositional formula* is either
(a) a simple formula
(b) a *propositional connective* applied to
the right number of arguments.

\wedge —
 \vee —
 \neg —
 \rightarrow —
 \leftrightarrow —

connectives

The language of logic



false

$x \in y \wedge x \geq 2$

$n \neq n \leftrightarrow (n = 1 \vee n = 3)$

Q: Is this formula True?

Truth tables

A	B	$A \wedge B$
True	True	True
True	False	False
False	False	False
False	True	False

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	False	True
False	True	True

A	B	$A \vee B$
True	True	True
True	False	True
False	False	False
False	True	True

A	B	$A \leftrightarrow B$
True	True	True
True	False	False
False	False	True
False	True	False

The language of logic: summary

Propositional formulae	Propositional connectives
Simple formulae	Predicate symbols
Terms	Constants, functions, variables

Truth tables define what the propositional connectives mean.

Q: Did the layered, systematic approach help against Russell's paradox?

Calculating with logic

Now we're here

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Calculating with logic

Propositional formulae

Are like the conditions in if-then-else, while

Terms

are like the RHS of assignment statements

We can *calculate* with logic as a thinking tool for programming,

...just as we can use mathematical calculation as a thinking tool for physics.

Calculating with logic

```
if l <= m < h:  
    ...  
else:  
    ... #what's true here?
```



Python syntax.

(let's calculate)

Calculating with logic

```
if l <= m < h:  
    thing1  
elif m < l:  
    thing2  
else:  
    thing3
```

```
if l <= m < h:  
    thing1  
elif m >= h:  
    thing3  
else:  
    thing2
```

Are these programs the same?

(let's calculate)